ESERCIZI SVOLTI
LOGARITMI

Determinare direttamente il valore dei seguenti logaritmi:

1) $\log_{11} 1 = x$ $\quad 11^x = 1$ $\quad 11^x = 11^0$ $\quad x = 0$.

2) $\log_6 6 = x$ $\quad 6^x = 6$ $\quad 6^x = 6^1$ $\quad x = 1$.

3) $\log_7 49 = x$ $\quad 7^x = 49$ $\quad 7^x = 7^2$ $\quad x = 2$.

4) $\log_3 27 = x$ $\quad 3^x = 27$ $\quad 3^x = 3^3$ $\quad x = 3$.

5) $\log_2 16 = x$ $\quad 2^x = 16$ $\quad 2^x = 2^4$ $\quad x = 4$.

6) $\log_2 32 = x$ $\quad 2^x = 32$ $\quad 2^x = 2^5$ $\quad x = 5$.

7) $\log_8 2 = x$ $\quad 8^x = 2$ $\quad 2^{3x} = 2$ $\quad 3x = 1$ $\quad x = \frac{1}{3}$.

8) $\log_6 36 = x$ $\quad 6^x = 36$ $\quad 6^x = 6^2$ $\quad x = 2$.

9) $\log_{36} 6 = x$ $\quad 36^x = 6$ $\quad 6^{2x} = 6$ $\quad 2x = 1$ $\quad x = \frac{1}{2}$.

10) $\log_9 81 = x$ $\quad 9^x = 81$ $\quad 3^{2x} = 3^4$ $\quad 2x = 4$ $\quad x = \frac{4}{2}$ $\quad x = 2$.

11) $\log_9 27 = x$ $\quad 9^x = 27$ $\quad 3^{2x} = 3^3$ $\quad 2x = 3$ $\quad x = \frac{3}{2}$.

12) $\log_{\frac{1}{3}} 3 = x$ $\quad \left(\frac{1}{3}\right)^x = 3$ $\quad \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-1}$ $\quad x = -1$.

13) $\log_{\frac{1}{5}} 25 = x$ $\quad \left(\frac{1}{5}\right)^x = 25$ $\quad \left(\frac{1}{5}\right)^x = 5^2$ $\quad \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-2}$ $\quad x = -2$.

14) $\log_{\frac{1}{5}} 125 = x$ $\quad \left(\frac{1}{5}\right)^x = 125$ $\quad \left(\frac{1}{5}\right)^x = 5^3$ $\quad \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-3}$ $\quad x = -3$.

15) $\log_{\frac{1}{3}} 27 = x$ $\quad \left(\frac{1}{3}\right)^x = 27$ $\quad \left(\frac{1}{3}\right)^x = 3^3$ $\quad \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-3}$ $\quad x = -3$.

16) $\log_{\frac{1}{2}} 64 = x$ $\quad \left(\frac{1}{2}\right)^x = 64$ $\quad \left(\frac{1}{2}\right)^x = 2^6$ $\quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-6}$ $\quad x = -6$.

17) $\log_{\frac{1}{2}} \frac{1}{2} = x$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{2}$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{2^1}$ $\quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^1$ $\quad x = 1$.

18) $\log_{\frac{1}{2}} \frac{1}{4} = x$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{4}$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{2^2}$ $\quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^2$ $\quad x = 2$.

19) $\log_{\frac{1}{7}} \frac{1}{49} = x$ $\left(\frac{1}{7}\right)^x = \frac{1}{49}$ $\left(\frac{1}{7}\right)^x = \frac{1}{7^2}$ $\left(\frac{1}{7}\right)^x = \left(\frac{1}{7}\right)^2$ $x = 2$.

20) $\log_{\frac{1}{2}} \frac{1}{128} = x$ $\left(\frac{1}{2}\right)^x = \frac{1}{128}$ $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^7$ $x = 7$.

21) $\log_{\frac{2}{3}} \frac{4}{9} = x$ $\left(\frac{2}{3}\right)^x = \frac{4}{9}$ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2$ $x = 2$.

22) $\log_{\frac{2}{3}} \frac{16}{81} = x$ $\left(\frac{2}{3}\right)^x = \frac{16}{81}$ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^4$ $x = 4$.

23) $\log_{\frac{2}{5}} \frac{25}{4} = x$ $\left(\frac{2}{5}\right)^x = \frac{25}{4}$ $\left(\frac{2}{5}\right)^x = \left(\frac{5}{2}\right)^2$ $\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^{-2}$ $x = -2$.

24) $\log_{\frac{3}{5}} \frac{125}{27} = x$ $\left(\frac{3}{5}\right)^x = \frac{125}{27}$ $\left(\frac{3}{5}\right)^x = \left(\frac{5}{3}\right)^3$ $\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^{-3}$ $x = -3$.

25) $\log_{\frac{9}{4}} \frac{3}{2} = x$ $\left(\frac{9}{4}\right)^x = \frac{3}{2}$ $\left(\frac{3}{2}\right)^{2x} = \frac{3}{2}$ $2x = 1$ $x = \frac{1}{2}$.

26) $\log_{\frac{49}{36}} \frac{6}{7} = x$ $\left(\frac{49}{36}\right)^x = \frac{6}{7}$ $\left(\frac{7}{6}\right)^{2x} = \left(\frac{7}{6}\right)^{-1}$ $2x = -1$ $x = -\frac{1}{2}$.

27) $\log_2 \sqrt{2} = x$ $2^x = \sqrt{2}$ $2^x = 2^{\frac{1}{2}}$ $x = \frac{1}{2}$.

28) $\log_2 \sqrt[3]{2} = x$ $2^x = \sqrt[3]{2}$ $2^x = 2^{\frac{1}{3}}$ $x = \frac{1}{3}$.

29) $\log_2 \sqrt[3]{4} = x$ $2^x = \sqrt[3]{4}$ $2^x = \sqrt[3]{2^2}$ $2^x = 2^{\frac{2}{3}}$ $x = \frac{2}{3}$.

30) $\log_3 \sqrt[3]{3} = x$ $3^x = \sqrt[3]{3}$ $3^x = 3^{\frac{1}{3}}$ $x = \frac{1}{3}$.

31) $\log_3 \sqrt[3]{9} = x$ $3^x = \sqrt[3]{9}$ $3^x = \sqrt[3]{3^2}$ $3^x = 3^{\frac{2}{3}}$ $x = \frac{2}{3}$.

32) $\log_{\frac{1}{3}} \sqrt[3]{9} = x$ $\left(\frac{1}{3}\right)^x = \sqrt[3]{9}$ $\left(\frac{1}{3}\right)^x = 3^{\frac{2}{3}}$ $\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-\frac{2}{3}}$ $x = -\frac{2}{3}$.

33) $\log_{\frac{1}{5}} \sqrt[3]{5} = x$ $\left(\frac{1}{5}\right)^x = \sqrt[3]{5}$ $\left(\frac{1}{5}\right)^x = 5^{\frac{1}{3}}$ $\left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-\frac{1}{3}}$ $x = -\frac{1}{3}$.

Dato il logaritmo e la base determinare il numero:

1)	$\log_3 x = 0$	$3^0 = x$	$x = 3^0$	$x = 1$.	
2)	$\log_3 x = 1$	$3^1 = x$	$x = 3^1$	$x = 3$.	
3)	$\log_3 x = 2$	$3^2 = x$	$x = 3^2$	$x = 9$.	
4)	$\log_3 x = 0$	$3^0 = x$	$x = 3^0$	$x = 1$.	
5)	$\log_2 x = 4$	$2^4 = x$	$x = 2^4$	$x = 16$.	
6)	$\log_8 x = 2$	$8^2 = x$	$x = 8^2$	$x = 64$.	
7)	$\log_{10} x = 2$	$10^2 = x$	$x = 10^2$	$x = 100$.	
8)	$\log_{10} x = 4$	$10^4 = x$	$x = 10^4$	$x = 10000$.	
9)	$\log_2 x = \frac{1}{2}$	$2^{\frac{1}{2}} = x$	$x = 2^{\frac{1}{2}}$	$x = \sqrt{2}$.	
10)	$\log_4 x = \frac{1}{2}$	$4^{\frac{1}{2}} = x$	$x = 4^{\frac{1}{2}}$	$x = \sqrt{4}$	$x = 2$.
11)	$\log_{27} x = \frac{1}{3}$	$27^{\frac{1}{3}} = x$	$x = 27^{\frac{1}{3}}$	$x = \sqrt[3]{27}$	$x = 3$.
12)	$\log_{\frac{1}{4}} x = -1$	$\left(\frac{1}{4}\right)^{-1} = x$	$x = \left(\frac{1}{4}\right)^{-1}$	$x = 4$.	
13)	$\log_4 x = -2$	$4^{-2} = x$	$x = 4^{-2}$	$x = \left(\frac{1}{4}\right)^2$	$x = \frac{1}{16}$.
14)	$\log_{11} x = -2$	$11^{-2} = x$	$x = 11^{-2}$	$x = \left(\frac{1}{11}\right)^2$	$x = \frac{1}{121}$.
15)	$\log_{\frac{1}{5}} x = 3$	$\left(\frac{1}{5}\right)^3 = x$	$x = \left(\frac{1}{5}\right)^3$	$x = \frac{1}{125}$.	
16)	$\log_{\frac{1}{4}} x = -3$	$\left(\frac{1}{4}\right)^{-3} = x$	$x = \left(\frac{1}{4}\right)^{-3}$	$x = 4^3$	$x = 64$.
17)	$\log_{\frac{1}{2}} x = -6$	$\left(\frac{1}{2}\right)^{-6} = x$	$x = \left(\frac{1}{2}\right)^{-6}$	$x = 2^6$	$x = 64$.
18)	$\log_{0,1} x = 2$	$\log_{\frac{1}{10}} x = 2$	$\left(\frac{1}{10}\right)^2 = x$	$x = \left(\frac{1}{10}\right)^2$	$x = \frac{1}{100}$.
19)	$\log_{0,2} x = 2$	$\log_{\frac{1}{5}} x = 2$	$\left(\frac{1}{5}\right)^2 = x$	$x = \left(\frac{1}{5}\right)^2$	$x = \frac{1}{25}$.
20)	$\log_{0,5} x = 3$	$\log_{\frac{1}{2}} x = 3$	$\left(\frac{1}{2}\right)^3 = x$	$x = \left(\frac{1}{2}\right)^3$	$x = \frac{1}{8}$.

Determinare la base dei seguenti logaritmi:

1) $\log_x 4 = 2$ $x^2 = 4$ $x^2 = 2^2$ $x = 2$.

2) $\log_x 9 = 2$ $x^2 = 9$ $x^2 = 3^2$ $x = 3$.

3) $\log_x 144 = 2$ $x^2 = 144$ $x^2 = 12^2$ $x = 12$.

4) $\log_x 27 = 3$ $x^3 = 27$ $x^3 = 3^3$ $x = 3$.

5) $\log_x 8 = 3$ $x^3 = 8$ $x^3 = 2^3$ $x = 2$.

6) $\log_x 1000 = 3$ $x^3 = 1000$ $x^3 = 10^3$ $x = 10$.

7) $\log_x 10000 = 4$ $x^4 = 10000$ $x^4 = 10^4$ $x = 10$.

8) $\log_x 32 = 5$ $x^5 = 32$ $x^5 = 2^5$ $x = 2$.

9) $\log_x 243 = 5$ $x^5 = 243$ $x^5 = 3^5$ $x = 3$.

10) $\log_x 64 = 3$ $x^3 = 64$ $x^3 = 4^3$ $x = 4$.

11) $\log_x 125 = 3$ $x^3 = 125$ $x^3 = 5^3$ $x = 5$.

12) $\log_x 64 = 6$ $x^6 = 64$ $x^6 = 2^6$ $x = 2$.

13) $\log_x 169 = 2$ $x^2 = 169$ $x^2 = 13^2$ $x = 13$.

14) $\log_x 2 = -1$ $x^{-1} = 2$ $x^{-1} = 2^1$ $x^{-1} = \left(\frac{1}{2}\right)^{-1}$ $x = \frac{1}{2}$.

15) $\log_x 16 = -4$ $x^{-4} = 16$ $x^{-4} = 2^4$ $x^{-4} = \left(\frac{1}{2}\right)^{-4}$ $x = \frac{1}{2}$.

16) $\log_x 8 = -3$ $x^{-3} = 8$ $x^{-3} = 2^3$ $x^{-3} = \left(\frac{1}{2}\right)^{-3}$ $x = \frac{1}{2}$.

17) $\log_x 25 = -2$ $x^{-2} = 25$ $x^{-2} = 5^2$ $x^{-2} = \left(\frac{1}{5}\right)^{-2}$ $x = \frac{1}{5}$.

18) $\log_x \sqrt{2} = \frac{1}{2}$ $x^{\frac{1}{2}} = \sqrt{2}$ $x^{\frac{1}{2}} = 2^{\frac{1}{2}}$ $x = 2$.

19) $\log_x \sqrt{3} = \frac{1}{2}$ $x^{\frac{1}{2}} = \sqrt{3}$ $x^{\frac{1}{2}} = 3^{\frac{1}{2}}$ $x = 3$.

20) $\log_x \sqrt[3]{2} = \frac{1}{3}$ $x^{\frac{1}{3}} = \sqrt[3]{2}$ $x^{\frac{1}{3}} = 2^{\frac{1}{3}}$ $x = 2$.

21) $\log_x \sqrt[5]{2^2} = \frac{2}{5}$ $x^{\frac{2}{5}} = \sqrt[5]{2^2}$ $x^{\frac{2}{5}} = 2^{\frac{2}{5}}$ $x = 2$.

22) $\log_x \sqrt[3]{\frac{1}{9}} = \frac{2}{3}$ $x^{\frac{2}{3}} = \sqrt[3]{\frac{1}{9}}$ $x^{\frac{2}{3}} = \left(\frac{1}{9}\right)^{\frac{1}{3}}$ $x^{\frac{2}{3}} = \left(\frac{1}{3}\right)^{\frac{2}{3}}$ $x = \frac{1}{3}$.